

Mr. Sraffa's Theory of Price; A Thorough and Critical Examination

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Abstract

Sraffa's *Production of Commodities by Means of Commodities* is critically examined without use of propositions peculiar to rival schools of thought. Where the subjective is obscured, it is exposed, including a fundamental presumption that is grossly unrealistic. A crucial dependence upon co-incident returns to scale is identified. A dire mathematic error is revealed. The method of sub-systems is shown to be fatally flawed. Other errors are brought to light. Sraffa's treatment of labor as a homogenous quantity is shown to be subject to a switching critique.

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Introduction

Although most American economists are unfamiliar with Piero Sraffa's *Production of Commodities by Mean of Commodities: Prelude to a Critique of Economic Theory*, it is the central text of contemporary neo-Ricardianism, one of the core texts of Post-Keynesianism, and the work embraced by those who would advance a variant form of Marxism that has stepped away from the labor theory of value. *Production of Commodities by Mean of Commodities* remains perhaps the most recent attempt of note to set aside subjectivist considerations and to explain prices and distribution in terms of objective relationships in productive processes. While *Production of Commodities* has itself been allowed to go out of print, copies of its various printings fetch significant premia, pirated electronic copies of an Indian edition circulate on the Internet, and new, expensive volumes of its secondary and tertiary literature are regularly published.

In this article, I examine the work, not from the perspective of an adherent nor through a lens of conclusions peculiar to rival schools of thought, but in terms of considerations intelligible to those without such commitments.

A Critical Walk-through

Both because a great share of economists are unfamiliar with Mr. Sraffa's book, and because the points about which to be concerned are many, this article will present almost a step-by-step explanation of Sraffa's presentation, along with commentary at many of these steps. While the reader's having a copy of *Production of Commodities* within reach would probably be desirable, I attempt to make discussion accessible to those who do not.

Sraffa's book has a preface, but neither an introductory nor concluding chapter to guide our interpretation of what he sets out to do or of what he believes himself to have accomplished. Attempting to address his over-all intentions is therefore a hazardous undertaking, which I have largely sought to avoid.

The "Subsistence" Economy

Sraffa's first model is of what he calls a "subsistence" economy.

Let us consider an extremely simple society which produces just enough to maintain itself. Commodities are produced by separate industries and are exchanged for one another at a market held after the harvest.[12, §1]

An economic system that produces just enough to maintain *itself* may produce rather more than enough for its agents to survive; however, in the course of later introducing a *third* model (to which we will come), Sraffa declares

We have up to this point regarded wages as consisting of the necessary subsistence of the workers and thus entering the system on the same footing as the fuel for engines or the feed for the cattle.[12, §8]

One of the peculiarities of *Production of Commodities by Mean of Commodities* is that it is very much concerned with mathematic relationships amongst production functions, yet Sraffa avoids explicit formal expression of these functions; he instead presents *snapshots* of production, with some rather awkward notation such as writing[12, §2]

$$240 \text{ qr. wheat} + 12 \text{ t. iron} + 18 \text{ pigs} \rightarrow 450 \text{ qr. wheat}$$

(which is intended to say that 450 quarters of wheat are produced from 240 quarters of wheat in combination with 12 tons of iron and 18 pigs — the ‘+’ plainly does not refer to arithmetic addition), and such as using ‘ A ’ for the quantity produced of the first sort of commodity and ‘ J_b ’ for that quantity of the tenth sort of commodity used in the production of the second commodity.[12, §3] I’ll take the liberty of writing

$$c_i(c_{1,i}, c_{2,i}, \dots, c_{k,i})$$

for the production function of the i th commodity, where ‘ $c_{j,i}$ ’ is quantity of the j th commodity input and k is the number of commodities, ‘ p_i ’ as the price of the i th commodity, and so forth. Sraffa explicitly denies making an assumption of “constant returns in all industries”, [12, Preface]

$$[\alpha \cdot c_i(c_{1,i}, c_{2,i}, \dots, c_{k,i}) = c_i(\alpha \cdot c_{1,i}, \alpha \cdot c_{2,i}, \dots, \alpha \cdot c_{k,i})] \forall (\alpha, c_i, c_{1,i}, \dots, c_{k,i}) .$$

(At various points, the reader should keep that *exact* denial in mind.)

(He implicitly imagines *technology* as something exogenous and typically fixed. He might instead have conceptualized it as a subset of the commodities. Some level of production would describe an economic system in which, for example, nothing were learned but nothing were forgot. Some technologic advancements might still need to be treated as windfalls.)

Since the quantities produced of each commodity in each period are exactly sufficient to produce the same quantities in the next, one may suppress indexing for periods, and simply write

$$\left[\sum_{j=1}^k (c_{i,j}) = c_i(c_{1,i}, c_{2,i}, \dots, c_{k,i}) \right] \forall i .$$

Sraffa asserts that, if these commodities are *priced*, the prices must be such that, for each commodity, the sum of input costs must equal the unit price of the commodity multiplied by the quantity produced,

$$\left[\sum_{j=1}^k (p_j \cdot c_{j,i}) = p_i \cdot c_i(c_{1,i}, c_{2,i}, \dots, c_{k,i}) \right] \forall i$$

and, if any one commodity is used as the standard of value, then all the other prices are mathematically determined.[12, §3]

His assertion follows from his assumptions. However, the formal *mathematics* at this stage permit an equilibrium in which *nothing* of anything is produced, in which case, *any* prices will do. From the *informal* description, we may infer that he means for there to be positive production of some commodities, whose prices will then be determined by the mathematics. This proposition means that agents are *motivated* to produce; one might ask *why*. If agents do not understand that “subsistence” depends upon them choosing precise prices, then they may very plausibly chose different prices that do not support a steady-state, perhaps even prices that lead them to the economic collapse of their society. But, without knowing rather more about the production functions, we cannot know whether all feasible deviations from the steady state cause a death spiral. Even if people do have such an understanding, they might decide that the requirements of existence were simply unacceptable. A condition

$$[c_i (c_{1,i}, c_{2,i}, \dots, c_{k,i}) \geq 0] \forall i$$

may be a statement about physical limitations, but the stronger condition

$$[c_i (c_{1,i}, c_{2,i}, \dots, c_{k,i}) > 0] \forall i$$

entails an assertion about choices, driven by *preferences*. Commodities are said to be produced by “industries”, but the decisions of industries are determined by human beings. Even if the decisions of each industry were determined by a computer program, human decisions — human preferences — would lie behind its design.

Instead of showing that the prices in this economic system are entirely determined by objective relations, Sraffa has implicitly *assumed* that the prices necessary for a positive steady-state can be reconciled with the preferences of the economic agents. One may grant that, for an economic system with only two possible long-run outcomes, one of which is basically *extinction*, if agents are *aware* of their predicament, then a rich description of preferences may not be needed to determine how things will be priced. However, Sraffa describes the produced commodities as exchanged in a “market”, and if this is not a mere pantomime then there must be an extraordinary co-incidence of preference with physical limitation.

(Consider an economic system without these physical limitations, but governed by occupiers who have decided, without informing their subjects, that the population will be exterminated if the market does not assign a particular set of prices to commodities. The survival of the populace implies *that* those prices were selected, but the harsh rule of the occupiers does not explain *why* they were selected.)

The Model with Surplus

In Sraffa's next model, the economic system produces more of some commodities than are consumed in production,[12, §4]

$$\left[c_i (c_{1,i}, c_{2,i}, \dots, c_{k,i}) \geq \sum_{j=1}^k (c_{i,j}) \right] \forall i$$

$$\wedge$$

$$\left[c_i (c_{1,i}, c_{2,i}, \dots, c_{k,i}) > \sum_{j=1}^k (c_{i,j}) \right] \exists i$$

Given a surplus of one or more commodity, there must be positive rates of profit in the determination of some prices,

$$\left[p_i \cdot c_i (c_{1,i}, c_{2,i}, \dots, c_{k,i}) = (1 + r_i) \cdot \sum_{j=1}^k (p_j \cdot c_{j,i}) \right] \forall i .$$

Instead of allowing for different rates of profit, Sraffa baldly states that the rate “must be uniform for all industries”,[12, §4]

$$\left[p_i \cdot c_i (c_{1,i}, c_{2,i}, \dots, c_{k,i}) = (1 + r) \cdot \sum_{j=1}^k (p_j \cdot c_{j,i}) \right] \forall i .$$

Many economists will be comfortable with that proposition, as it represents a simplifying limiting case. However, it is not a necessary expression of economic law; it entails (amongst other things) assumptions about preferences concerning the source of one's income. That asserted uniformity gives him as many unknowns as equations, but it is either quite arbitrary or makes yet another assumption about preferences. The constraint corresponds to a presumption that whatever entities are making productive decisions are pecuniary profit maximizers, and that no agent accepts a lower rate of pecuniary profit because of some other perceived benefit.

The surplus is introduced into this model by assuming that one or more of the production functions is more productive at the same level of inputs that was assumed for the previous model; but, unless the relevant production processes use unique inputs in fixed proportions, there ought to be multiple possible outcomes, amongst which agents choose. Sraffa implicitly assumes some levels of inputs to each process and thus some level of output for each commodity, and then offers an ostensible explanation of prices as a solution to the system of those equations that remain after these choices.

As a consequence of the surplus, Sraffa distinguishes what he calls “basic” commodities from what he variously calls “non-basic” products or “luxury” products. The former are returned, directly or indirectly, as inputs in the production of all commodities, the latter are not (and are either not returned as inputs at all, or serve only as inputs for other non-basic products).[12, §6]

Later, Sraffa revisits the concept of a basic commodity, and under a revised conception will arrive at a conclusion that *land* cannot be a basic commodity, because nothing enters into its production.[12, §§57–60] But we should see that, under this initial definition of “basic”, land cannot be a commodity *at all* if any commodities are to be basic, given that nothing enters into the production of land and that any basic commodity must enter into the production of every commodity.

He asserts

If an invention were to reduce by half the quantity of each of the means of production which are required to produce a unit of a ‘luxury’ commodity of this type, the commodity itself would be halved in price, but there would be no further consequences; the price-relations of the other products and the rate of profit would remain unaffected. But if such a change occurred in the production of a commodity of the opposite type, which *does* enter the means of production, all prices would be affected and the rate of profit would be changed.[12, §7]

(However, only if returns were co-incident or consumption were unit-elastic would a halving of costs at each level of production imply that the new price of the ‘luxury’ commodity would be half that of the old price. And the rate of profit is unaffected simply because it is exogenous *by assumption*.) Sraffa thus proposes

to explain why the ratios which satisfy the conditions of production have been called ‘values’ or ‘prices’ rather than, as might be thought more appropriate, ‘costs of production’.[12, §7]

He writes

[T]he price of a non-basic product depends on the prices of its means of production, but these do not depend on it. Whereas in the case of basic products the prices of its means of production depend on its own price no less than the latter depends on them.[12, §7]

A distinction between exchange ratios determined with and without feedback seems reasonable enough; but his claim that a change in the production function of non-basic goods will not affect the ratios of basic goods is entirely too fragile. Allowing that technical exchange ratios happen to be chosen as prices, his assertion will not hold if a surplus can be applied to increase production of basic commodities except, perhaps, when returns are co-incident (as with constant returns), or if the non-basic commodities are so *desired* that the surplus will always instead be applied to their production. (In other words, there must be a remarkable technical constraint, or preferences incompatible with investment to increase capacity.) Otherwise, a change in the production function for the non-basic goods can cause a change in allocations that disturbs the production levels and the associated exchange ratios. The slightest move towards realism

invalidates his conclusion. Much later, he offers an example in which tractors are produced in increasing quantity;[12, §81] tractors seem a basic commodity, and it appears that returns in the production of tractors must be at least co-incident if not indeed simply constant.

The Model with a Separated Labor Component

Having apparently satisfied himself that some commodities are epiphenomenal, with prices that do not figure in the determination of the prices of basic commodities, Sraffa proposes to treat *labor* as if a formula determining its price need not appear in his systems of equations. He acknowledges that commodities to sustain workers are “essentially basic”, but proposes to account for this “in devious ways”, giving the example of “setting a limit below which the wage cannot fall”. [12, §8] It should be noted that, *by treating the sustenance of labor as if it were non-basic, Sraffa has categorized it as a mathematic component of the surplus.* (In describing Sraffa’s model, Hahn declares “We also think of wages as payments over and above subsistence.” [8, p 356] This interpretation does not conform to Sraffa’s prose, nor do the pricing formulæ make any allowance for production that will provide for workers as such other than through wages.)

And, although he acknowledges that some of the surplus of the economic system may go to labor in the form of a wage greater than subsistence, he takes the total annual labor of society to be unity, [12, §10] which quietly implies that the quantity of labor is not variable, as if workers supply just as much labor under the best of circumstances as they would under the most desperate. Discussion of how the model *might* be modified to ensure subsistence for the workers is of little use if one proceeds, as does Sraffa, without making those modifications and treats the case in which workers as such are paid nothing as meaningful. Only if the workers are also rentiers in these models can they survive given a wage rate of 0; and, given the constancy of the quality and quantity of labor, for the workers to be rentiers would make a mockery of the relationships that Sraffa derives amongst wages, profits, and commodity prices. Of course, it likewise is only if the rentiers are also laborers that they can survive given a profit rate of 0. But Sraffa altogether avoids a discussion of *ownership*.

Sraffa’s third model separates labor from other productive resources. He writes

We suppose labour to be uniform in quality or, what amounts to the same thing, we assume differences in quality to have been previously reduced to equivalent differences in quantity so that each unit of labor receives the same wage. [12, §10]

I will give more attention to this supposition in the context of considering the implications for capital theory of switching methods of production.

His equations of price are now

$$\left[(1+r) \cdot \sum_{j=1}^k (p_j \cdot c_{j,i}) + w \cdot L_i = p_i \cdot c_i(c_{1,i}, c_{2,i}, \dots, c_{k,i}, L_i) \right] \forall i ,$$

where w is the wage rate and L_i is the amount of labor devoted to the production of the i th commodity. (Sraffa instead uses ‘ L_a ’, ‘ L_b ’, &c.)[12, §11] Workers are presumed to have no *preferences* for working in some industries as opposed to others (which preferences could result in distinct w_i), though Sraffa does not make this claim explicit.

Note that any other means of production — perhaps a basic commodity, or perhaps some intermediate product — could have been treated as if its inputs were non-basic, as if its supply were inelastic, and as if it would naturally command the same price in any occupation, and would then have the same systemic properties as these assumptions give to labor.

And note that, once labor is treated in this way, commodities are no longer acknowledged as indirect inputs to the production of other commodities by way of their effect on the quantity of labor, and what had been basic commodities can become partitioned into separate spheres of recognized feedback. When Sraffa, in the course of describing what he called a “subsistence” economic system, made the assumption that any system would have at least one basic commodity,[12, §6] that assumption was fairly natural; in the context of treating labor as he now does, the assumption becomes rather *ad hoc*.

The Standard of Value and the Wage Rate

With the third model constructed, Sraffa proposes to use surplus production to establish a *standard of value*, according to the equation[12, §12]

$$\sum_{i=1}^k \left(p_i \cdot \left[c_i - \sum_{j=1}^k (c_{i,j}) \right] \right) = 1 . \quad (1)$$

Of course, in the absence of a surplus, this standard becomes absurd. However, Sraffa has chosen this particular standard of value to exhibit a relationship between the wage rate and the rate of profit.

Specifically, “on the assumption that the methods of production remain unchanged”, when the wage w is equal to 1, then the whole of the surplus is absorbed by wages, and the rate of profit r must be 0; when w is 0, r is at a maximum. If one loses sight of the fact that Sraffa has begun treating the subsistence of the worker as a component of the surplus, and that labor supply is presumed to be perfectly inelastic, then this complementary relationship might seem profound. Removing the artefacts of modelling, one is left with the insight that some of what might otherwise be collected as profit could instead be paid as wages or *vice versa*. (The same is of course true for other means of production.) Note that either the aforementioned odd technical constraint on investment obtains, or the over-all propensity to invest is somehow constant, regardless of how much is paid in wages and how much in profits.

In what seems a nod to the labor theory of value, Sraffa asserts that, when surplus production is directed entirely to wages, “the relative values of commodities are in proportion to their labour costs, that is to say to the quantity of labour which directly and indirectly has gone to produce them.”[12, §14] He

argues for this conclusion by imagining the system of industries divided into *sub-systems*, with as many such sub-systems as commodities in the net product, and just one such commodity in each sub-system. All production goes either directly into production of the surplus commodity or into replacement of the means of production for that surplus commodity. At the end of a productive cycle, one has the same amount of commodities for the next productive cycle, plus the net product, with a net expenditure only of labor; so, as he would have it, the wage cost is the sole cost and must then be the price of the net product.[12, App A]

But consider the pricing of a commodity that were not produced in surplus, as could happen in these models with a purely infrastructural commodity. A sub-system for this commodity would have a net production of 0, but these models have all presumed a need for active renewal, so there would be an expenditure of labor. If returns to scale were co-incident, so that the sub-system might be embodied, that sub-system standing alone would produce a wage of 0, but wages are presumed to be shared across sub-systems (else Sraffa's argument falls apart). Hence, it appears that prices of commodities of this infrastructural sort must be whatever one makes of division by 0. The underlying problem (as I will explain more fully in a later section) is that he has lost sight of the fact (more obvious in the context of a subsistence-level economic system) that when all resources are idled, some begin to perish; renewed or replaced commodities must be counted as co-products.

Thus it may be seen that the odd labor supply function of Mr Sraffa's models is not merely unrealistic, but has facilitated an implicit self-contradiction. This contradiction is not removed by imagining (as does Hahn[8, p 356]) that the wage is an amount paid above subsistence, as the wage rate has been presumed to be uniform across employments. Nor may the the significance of wages paid in basic industries be treated as negligible unless the basic commodities are a tiny share of production and the potential rate of profit is many thousand percent. The contradiction is removed if the labor pool is partitioned, with workers in basic industries laboring for nothing. Mr Sraffa later twice uses his method of sub-systems to draw involved conclusions, himself laboring as if basic industries paid a wage of 0 even while others pay a positive wage.[12, §§66–72, §§80–84]

Sraffa notes that if (as will surely be the case) some industries are, directly or indirectly, more labor intensive than are others, then a change in the wage rate w will differently affect prices of different commodities. He would like a numéraire itself unaffected by changes in w . Drawing upon propositions that in turn lean upon the unacknowledged or obscured assumptions that have been noted above, he arrives at a description for an industry in which wages and profits are exactly “balanced”, so that associated profits rise or fall in a manner that exactly offsets what would otherwise be the effect of a change in the wage rate w . [12, §§15–22]

Sraffa acknowledges that such a commodity is unlikely to be produced by any actual industry, but proposes to construct a composite *Standard commodity* to serve as a standard of value.[12, §§23–26] Sraffa does not give this commodity

a mathematic symbol; I'll call it

$$\mathbf{c}_0 = [c_{1,0}, c_{2,0}, \dots, c_{k,0}]$$

where $c_{i,0}$ is the amount of the i th commodity in the composite. He writes “the perfect composite commodity of this type [...] is one which consists of the same commodities (combined in the same proportions) as does the aggregate of its own means of production”, [12, §24]

$$\mathbf{c}_0(c_{0,0}, L_0) = (1 + \hat{r}) \cdot \mathbf{c}_{0,0} ,$$

where $\mathbf{c}_{0,0}$ is the amount of this Standard commodity applied to the further production of the Standard commodity,

$$L_0 = \sum_{i=1}^k \left(\frac{c_{i,0}}{c_i} \cdot L_i \right) ,$$

and the rate of increase \hat{r} is what Sraffa later calls the “Standard ratio”. [12, §28, §30] Amongst the number of confusions in Sraffa’s exposition, one should perhaps be mentioned immediately; while Sraffa means to consider Standard commodities whose Standard ratios equal the maximum rate of profit R , he has not *defined* “Standard ratio” thus; his later exposition confuses this matter. [12, §30, esp §33ff]

By “Standard system”, he means the combination of industries and (isomorphically) the system of equations that respectively generate or identify the Standard commodity. (Sraffa refers to the more general model, the product of which may not be completely represented in terms of a Standard commodity, as an “actual” economic system. [12, §25ff]) The construction of a Standard system is largely to create a case in which the Standard ratio would be the maximum rate of profit, [12, §30] exactly because it is not definitionally so.

(Through-out *Production of Commodities by Mean of Commodities*, Sraffa constructs and reconstructs Standard systems. I do not see good reason for him ever to proceed on the assumption that a Standard system may be embodied; for what I take to be their purpose, he merely needs proof of existence of an abstract sort; but he repeatedly treats Standard systems as things that might be given physical existence, though he concedes that this embodiment may be impossible in the case of multi-product industries. [12, §56] In any case, embodiment repeatedly entangles him in unacknowledged issues of co-incident economies of scale.)

Using a system in which all commodities are basic and in which every production function implicitly has co-incident returns, Sraffa illustrates construction of a Standard commodity. He reduces inputs, by the same proportion for any given production function though by possibly different proportions across industries, until what is left is a Standard system. [12, §25] He later declares “It can be said that in any actual economic system there is a miniature Standard system which can be brought to light by chipping off the unwanted parts.” [12, §26] But in an economic system in which production functions do not have constant returns,

it might be that “chipping” ultimately left one with nothing; if chipping did not, then without at least co-incident returns there would still be a distinction between a composite conforming to the production functions of the model and one conforming to functions with constant returns intersecting those production functions at the present levels of production.

Sraffa chooses a *unit* of this commodity to be “the quantity of it that would form the net product of a Standard system employing the whole annual labour of the actual system”. The unit is called “the Standard net product” or “Standard national income”. If one uses a Standard commodity each of whose components have production functions with co-incident returns to scale, then this unit could be computed by multiplying each component in the Standard commodity by L/L_0 , just as Sraffa does,[12, §26] and its \hat{r} would be unchanged. With other than co-incident returns, it might be that what were produced would not even be expressible in terms just of the Standard commodity; and an unchanged \hat{r} should come as a surprise.

Sraffa has assumed co-incident returns to scale — not of *all* commodities, but of all *basic* commodities — in order to get the Standard system as an embodied economic system. But, for the ultimate purpose of finding the numéraire that he desires, he doesn’t actually need an economic system; he only needs a basket such that, given prevailing ratios between factors and output, the amount of production corresponding to the basket represents an increase in each commodity in the basket by a ratio equal to the maximum rate of profit R . And, mathematically, that’s equivalent to identifying a Standard system with constant returns to scale which has a Standard ratio of R . Multiplication of each component in the basket by L/L_0 is then simply a normalization.

Sraffa shows how he would find a Standard system with a Standard ratio equal to the maximum rate of profit of a non-Standard system with the same production functions. Specifically, he would find multipliers q_i such that

$$\left[(1 + R) \cdot \sum_{j=1}^k (q_j \cdot c_{i,j}) = q_i \cdot c_i (c_{1,i}, c_{2,i}, \dots, c_{k,i}) \right] \forall i$$

$$\wedge$$

$$\left[\sum_{j=1}^k (q_j \cdot L_j) = L \right]$$

(with L being normalized to 1).[12, §33] Sraffa declares this to be a system of $i + 1$ equations and $i + 1$ unknowns, the unknowns being the q_i and R . [12, §33] Treating R as an unknown implies that the maximum rate of profit in the more general model is somehow preserved in this discovered basket.

For it to have the properties that Sraffa desires for his a numéraire, it must meet the condition

$$\left[q_i \cdot \left([1 + r] \cdot \left[\sum_{j=1}^k (p_j \cdot c_{i,j}) \right] + L_i \cdot w \right) = q_i \cdot p_i \cdot c_i (c_{1,i}, c_{2,i}, \dots, c_{k,i}) \right] \forall i.$$

(Sraffa unhelpfully attempts to distinguish the values found for these multipliers by adding prime marks to the symbols for the multipliers. This distinction is without a difference.)[12, §34]

Unfortunately for the purpose of constructing a Standard system equivalent to the more general model, or that of simply identifying a basket to serve as numéraire, this method is somewhat fragile. Consider the simple production functions

$$\begin{aligned} c_1(c_{1,1}, L_1) &= 1.5 \cdot \min\{c_{1,1}, \kappa_{L,2} \cdot L_1\} \\ &\wedge \\ c_2(c_{1,2}, c_{2,2}, L_2) &= 1.35 \cdot \min\{10 \cdot \kappa_{1,2} \cdot c_{1,2}, c_{2,2}, \kappa_{L,2} \cdot L_2\} \end{aligned}$$

in which the various κ are conversion factors (ratios of units of one commodity or of labor to units of another commodity). If $c_{1,2} = c_{1,1}/9$ with $c_{2,2} = (c_{1,1} + c_{1,2})/\kappa_{1,2}$, then both $c_1(c_{1,1}, L_1)/c_{1,1}$ and $c_2(c_{1,2}, c_{2,2}, L_2)/c_{2,2}$ equal 1.35, and indeed Sraffa's method of finding the Standard system gives $R = .35$. But the value for q_1 or that for q_2 must be *negative*.

Sraffa acknowledges the existence of such cases, and identifies them with non-basic commodities that happen to have a net production equal to the maximum rate of profit; he classifies these as “freak” cases.[12, §35 fn 1, App B] However, a *real-world* economic system, of very many different commodities, will have freakish cases of various sorts. (It would be freakish if it did not.) It should not be at all surprising to find at least one commodity whose ratio of net product just happens to equal an overall net production rate or maximum rate of aggregate profit, should such a rate or rates be meaningful in the first place. And just one is all that is needed to bring ruin to the method of identification. Sraffa does not tell us what to do in such cases. If c_2 and its factors above are omitted from the calculation (on an *ad hoc* basis or otherwise) then R would be computed as .5, whereas $[1 - c_i(c_{1,i}, c_{2,i}, L_i)/c_{i,i} = .35] \forall i$.

Because Sraffa has assumed the presence of at least one basic commodity (a direct or indirect factor in the production of *every* commodity).[12, §6] any Standard commodity is necessarily a composition including some positive amount of each such commodity; and any commodity that contributes to the production of a basic commodity is itself *ipso facto* a basic commodity (as it must be at least an indirect factor in the production of all other commodities). Thus, if a Standard commodity (with $\hat{r} = R$ or otherwise) exists then one can be constructed of basic commodities.

In cases in which the net production rate of a non-basic commodity does not equal R , the multiplier q_i found for it will be 0;[12, §35] so the Standard system will produce only basic commodities or the basket identified as numéraire will consist only of basic commodities.

Again implicitly presuming co-incident rates of return, Sraffa argues that, for any more general system, there will always be at least one corresponding Standard system,[12, §37] and that not more than one Standard system will correspond to exclusively positive prices, this system then uniquely identifying the maximum rate of profit R ,[12, §§38–41] quietly excepting of course the cases in which one or more non-basic commodity has a net production rate

equal to the maximum rate of profit R . Based upon this near-proof that there uniquely exists a Standard system corresponding to exclusively positive prices, he concludes that, so long as prices are found such that

$$r = R \cdot (1 - w) ,$$

these prices are implicitly in terms of the Standard net product. (I omit an unhelpful prime mark.) R may be found from the production equations by setting $w = 0$. Thence, Sraffa suggests that prices might be measured not in terms of the Standard net product, but in terms of the amount of labor that may be purchased with the Standard commodity. With the total quantity of labor normalized to 1, this equals

$$\frac{1}{w} = \frac{R}{R - r} ,$$

rising without limit as the wage rate drops to 0, which is to say as r approaches R . [12, §43] (Had he similarly assumed some other factor to be fixed, then prices might be measured in terms of the amount of that factor that could be purchased with the Standard commodity.)

Sraffa obliquely acknowledges that he *cannot* explain r at all within his model (except to say that it mustn't be greater than R and perhaps that it won't be less than 0); he treats this limitation of his model as license to assert that r is a rate of interest determined by "the level of the money rates of interest", [12, §44] though why, in the sort of economic system that he describes, interest rates should determine anything more than how profits are differently shared amongst the rentiers is not explained.

Reduction to Dated Quantities of Labor

Assuming constant prices, wages, and profits, Sraffa proposes that, beginning with

$$p_i \cdot c_i (c_{1,i}, c_{2,i}, \dots, c_{k,i}, L_i) = w \cdot L_i + (1 + r) \cdot \sum_{j=1}^k (p_j \cdot c_{j,i}) ,$$

one may recursively replace each commodity by its own means of production, to arrive at an infinite series

$$p_i \cdot c_{t,i} = \sum_{j=0}^{\infty} \left[(1 + r)^j \cdot w \cdot L_{t-j,i} \right] ,$$

where the additional subscripts t and $t - k$ index the period in which the commodity were produced or the labor employed. [12, §46] Sraffa's verbal explanation is loose and seemingly casual, and he does not present formal mathematic expression of the steps by which he arrives at this expansion, but one should see that it is incorrect; under the assumptions by which he arrived at the formula from which he here proceeds, the surplus production of each round is not

given to production in the next round. Thus, the full factor cost of commodities produced in previous rounds is not implicitly present in production of the next round; and, in particular, the full labor cost is not present. In the simplest case, the expansion conforming to his prior assumptions would be

$$p_i \cdot c_{t,i} = \sum_{j=0}^{\infty} \left[\left(\frac{1+r}{1+R} \right)^j \cdot w \cdot L_{t-j,i} \right],$$

with $r < R$.

(Were expansions such as his correct, and had some other factor been uniquely assumed to have constant quantity regardless of price, then the expansions could have been in terms of that factor.)

Substituting from

$$w = 1 - \frac{r}{R},$$

Sraffa arrives at

$$(1+r)^j \cdot \left(1 - \frac{r}{R} \right) \cdot L_{t-j,i}$$

for the j th term,[12, §47] whereas what he should have is

$$\left(\frac{1+r}{1+R} \right)^j \cdot \left(1 - \frac{r}{R} \right) \cdot L_{t-j,i}.$$

With the division by $1+R$ overlooked, Sraffa writes “At $r = 0$ the value of a labour term depends exclusively on its size, regardless of date.”[12, §47] Instead, however, each term becomes

$$\frac{1}{(1+R)^j} \cdot L_{t-j,i},$$

with the value of each term shrinking as it becomes more distant. Indeed, *an infinite number of constant, finitely positive terms would sum to infinity.*

From his mistaken formula, Sraffa draws some conclusions about relationships amongst wages, prices, and profits. He offers a particular example of two commodities which differ in just three of their labor terms.[12, §48] One had 20 more units of labor applied than the other 8 years ago; the other drew 1 more unit 25 years ago and 19 more units in the present year. He concludes that their prices must differ by

$$\left[20 \cdot w \cdot (1+r)^8 \right] - \left[19 \cdot w + w \cdot (1+r)^{25} \right]$$

but he should have arrived at the formula

$$\left[20 \cdot w \cdot \left(\frac{1+r}{1+R} \right)^8 \right] - \left[19 \cdot w + w \cdot \left(\frac{1+r}{1+R} \right)^{25} \right]$$

Supposing a maximum rate of profit of 0.25, his formula becomes

$$\left(1 - \frac{r}{.25} \right) \cdot \left(\left[20 \cdot (1+r)^8 \right] - \left[19 + (1+r)^{25} \right] \right)$$

when it ought instead to be

$$\left(1 - \frac{r}{.25}\right) \cdot \left(\left[20 \cdot \left(\frac{1+r}{1.25}\right)^8 \right] - \left[19 + \left(\frac{1+r}{1.25}\right)^{25} \right] \right)$$

His formula gives the price difference of 0 when r is 0, about 0.170934, or 0.25, being positive in the lower open interval and negative in the upper open interval. His example is apparently intended in part to illustrate a claim about measures of capital:

The reversals in the direction of the movement of relative prices, in the face of unchanged methods of production, cannot be reconciled with *any* notion of capital as a measurable quantity independent of distribution and prices.[12, §48]

The reversal in his example is an artefact of a serious error on his part. With these values for labor-input differences, the corrected formula will be 0 when r is 0.25; it is negative everywhere in the interval $[0, 0.25)$. However, selecting different values for labor-input differences that will, with the corrected formula, exhibit the sort of reversal that Sraffa sought to show is quite possible.

At this point, recall his earlier supposition that labor were effectively uniform in quality and fixed in quantity employed.[12, §10] In reality, it is neither, though its pricing in terms of a common currency might mislead one with respect to effective uniformity, much as might the pricing of capital in a common currency. To generalize Mr Sraffa's models to acknowledge the heterogeneity, one would distinguish amongst methods of production by the sorts of labor that they used, there would be different wages rates for different sorts. As r varied (presuming that it continued to be exogenously determined), there would be switches in the methods of production, and *these wages rates would not simply maintain fixed ratios each to the others*. Mr Sraffa's illustration of relative price movements of capital of different sorts employs his supposition of effective uniformity and fixed employment of labor; an illustration of relative prices movements of labor of different sorts would be rather messy unless one analogously supposed there to be just one sort of capital, employed in constant quantity. But the point should be obvious.

Sraffa additionally argues that when prices fall in response to a rise in the rate of profit r , they cannot fall faster than does the wage rate w , so that a price can equal the wage rate at most once. Thence he argues that a cut in the wage rate in terms of any commodity must necessarily imply an increase in the rate of profit and thus a wage cut in terms of all commodities.[12, §49] However, he later rejects these relationships (along with the whole business of Reduction!) in the cases in which single processes may produce more than one commodity.[12, §§69–72]

The Model with Multi-Product Industries

Sraffa next proposes to abandon the assumption that each productive process generates just one product. (I'll represent the generalization with a vector func-

tion

$$[\hat{c}_{1,j}, \hat{c}_{2,j}, \dots, \hat{c}_{k,j}] = \hat{c}_j (c_{1,j}, c_{2,j}, \dots, c_{k,j})$$

where ‘ $c_{i,j}$ ’ now represents the amount of the i th commodity used as a factor in the j th function, and ‘ $\hat{c}_{i,j}$ ’ is the amount of that commodity generated by that function.) Simply reducing the number of production functions to less than the number of commodities would leave prices indeterminate; Sraffa proposes to assume that each product has more than one productive process, to keep the number of commodities and the number of production functions equal.[12, §50] On the assumption that commodities will indeed be priced, an assumption that there will be as many equations as values to be determined is fair enough. But the presumption that these equations will be fundamentally technical ought to have been given some justification.

Sraffa concedes that, with multi-product processes, it may not be possible to construct a Standard commodity all of whose components are positive quantities. The Standard system can then no longer be imagined as having a bodily existence. He is, however, at this point willing to conceptualize the Standard commodity as an imagined basket of assets and of liabilities, rather than as the product of a reconstructed economic system.[12, §§53–56]

Under the initial definition of “basic”, if there is *any* amount of some product for which no amount of a commodity is a factor (directly or indirectly), that commodity is non-basic, even if there is some other amount to which the commodity did contribute. Multi-product industries may not use factors of the same *sorts* in all processes yielding some common product, and thus what seems, from regarding one process, to be a basic commodity may be discovered not to be upon regarding another.[12, §57] Sraffa proposes to revise the definition of “basic” by considering equivalents in systems of multi-product industries to the sorts of *non*-basic commodities identified in systems of single-product industries, and then to define “basic” as the negation of the new notion of *non*-basic. Commodities that are never factors remain a possibility. Next were products that never entered into any productive process except each for itself; he asserts that the equivalents are commodities such that the ratio of the quantities used as factors to the quantities produced are the same across all processes. The third original sort of non-basic commodities were factors of production for other non-basic commodities, but these were never themselves in turn basic; he sees the equivalent in the set of commodities whose values both as inputs and as outputs can be reduced to 0 after forming a matrix whose columns correspond to each sort of commodity, once as input and once as output, and whose rows correspond to each productive process,

$$\left[\begin{array}{cccc|cccc} \hat{c}_{1,1} & \hat{c}_{2,1} & \cdots & \hat{c}_{k,1} & c_{1,1} & c_{2,1} & \cdots & c_{k,1} \\ \hat{c}_{1,2} & \hat{c}_{2,2} & \cdots & \hat{c}_{k,2} & c_{1,2} & c_{2,2} & \cdots & c_{k,2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{c}_{1,k} & \hat{c}_{2,k} & \cdots & \hat{c}_{k,k} & c_{1,k} & c_{2,k} & \cdots & c_{k,k} \end{array} \right]$$

and then subtracting rows. This notion in fact subsumes the immediately previous two sorts of non-basic commodities.[12, §§58–61] In other words, a com-

modity is now said to be “basic” if it corresponds to a variable that cannot be algebraically eliminated from a system of equations corresponding to the production of an economic system. It should be noted that, in replacing the original notion of a “basic” commodity, he has implicitly replaced the earlier assumption that there were at least one commodity of the original sort with an assumption that there be at least one commodity of this new sort.

The new definition for “basic” has the result of rendering *land*, which may enter into every productive process but does not itself result from such a process, as non-basic.[12, §60] (As I noted above, under the initial definition of “basic”, land could not be a commodity at all if there were to be any basic commodities. Labor was subsequently rendered non-basic, and scarcely if at all even acknowledged to be a commodity.)

Sraffa uses the term “Basic equations” to refer to the system of equations obtained when the values for non-basic commodities are algebraically reduced to zero, and the values in the resulting matrix,

$$\left[\begin{array}{cccc|cccc} \hat{g}_{1,1} & \hat{g}_{2,1} & \cdots & \hat{g}_{k,1} & g_{1,1} & g_{2,1} & \cdots & g_{k,1} \\ \hat{g}_{1,2} & \hat{g}_{2,2} & \cdots & \hat{g}_{k,2} & g_{1,2} & g_{2,2} & \cdots & g_{k,2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{g}_{1,k} & \hat{g}_{2,k} & \cdots & \hat{g}_{k,k} & g_{1,k} & g_{2,k} & \cdots & g_{k,k} \end{array} \right]$$

(some of whose $\hat{g}_{i,j}$ and $g_{i,j}$ are presumably 0), are entered into the pricing equations

$$\sum_{j=1}^k (p_j \cdot \hat{g}_{j,i}) = w \cdot L_i + (1+r) \cdot \sum_{j=1}^k (p_j \cdot g_{j,i}) .$$

He warns “a Basic equation does not in general represent a productive process”. [12, §62] In fact, it could only do so in the case of a process that could take place in the vacuum of outer space. He also notes that “it may contain negative quantities as well as positive ones”, [12, §62] though this seems unremarkable to me, as negative outputs are disguised inputs, and negative inputs are disguised outputs. But indeed the Basic system is plainly very different from the practical core of a real economic system.

The Basic system is in part a conceptual way-station to a construction of a Standard commodity, [12, §63 fn 1, App C]

$$\sum_{j=1}^k (q_j \cdot \hat{g}_{j,i}) = w \cdot L_i + (1+r) \cdot \sum_{j=1}^k (q_j \cdot g_{j,i}) ,$$

with the q_i selected to produce the lowest value of maximum profit R , in an attempt to avoid the absurdity of infinite prices. [12, §64]

Within the context of his earlier models in which each productive process has just one sort of output, Sraffa reaches the fragile conclusion that the prices of non-basic commodities did not affect the prices of basic commodities. And, before he takes wages as drawn against the surplus (so that $r = R$),

the production of basic commodities determines the rate of profit.[12, §6] In the case of his model with processes that have more than one sort of output, he asserts that the price of non-basic goods and in particular a tax on non-basic goods cannot affect the price of basic goods nor the rate of profit.[12, §65] This claim confuses the rate of profit r with the *maximum* rate of profit R ; the rate of profit r is determined in a mysterious money market.[12, §44] Stronger assumptions about that market are required to prevent a tax on non-basic goods from affecting r and hence the wage rate w , a point that has later significance. Otherwise, the conclusion does flow from his assumptions, but implausible assumptions about technical constraints or about preferences are still involved.

To show that, even in the case in which processes produce more than one commodity, the relative value of each commodity remains proportional to the quantity of labor used in production when the rate of profit is 0, Sraffa proposes again to argue in terms of sub-systems.[12, §§66–67] I again note that this method fails to account for the costs of renewing or preserving perishable resources. He rejects the use of Reduction to a series of wage costs on the grounds that negative coefficients will be needed for some of the joint products, which in turn will imply negative quantities of labor; he cannot see an interpretation for these, and infers that their presence might imply divergent series.[12, §68] I think that the latter concern may be misplaced, as he has forgot to discount for the extraction of the surplus in his formulæ for Reduction (see above), but I have not investigated. He notes that some commodities may, as a result of a change in the wage rate, begin to have negative prices and yet continue to be produced so long as these negative prices are offset by a rise in the price of their coproducts.[12, §69] The ordinary interpretation would be that what were economic goods may become waste products in the production of other goods, yet Sraffa suggest that the presence of negative prices might seem “a freak result of abstraction-mongering that can have no correspondence in reality”, and he argues for the plausibility of the result in terms of net pecuniary profit.[12, §70] Sraffa mounts a *separate* argument that the prices of some commodities may fall faster than the wage rate,[12, §71] but the line of reasoning that allows some prices to go from positive to negative implies that much along the way. Sraffa labors the point that, when some prices fall in response to a change in the wage rate while others rise in response to a change in the wage rate, the wage itself may have risen in terms of one price and fallen in terms of the other. But he asserts that, however the wage rate might be measured, there will be exactly one corresponding rate of profit.[12, §71] That bijection remains an artefact of such assumptions as the invariable employment of labor.

The Model with Fixed Capital

Sraffa’s primary purpose in considering multi-product industries is to support a method of considering capital goods that will somewhat endure across productive cycles. These goods are considered to become different commodities as they wear from use. Thus, what might otherwise be considered a process producing

m different commodities and using n durable capital goods is instead considered to produce $m + n$ commodities.[12, §73] And if an industry does not replace the same shares of its capital goods in each productive period, then it corresponds to a *sequence* of productive processes across periods, rather than just to one process repeated in each productive period.[12, §74] (Lengthening the spans regarded as periods of production could exhaust durable capital and reduce sequences to the length of one period; but with capital wearing at different rates the length of a sequence might have to be very long, in that otherwise some sequences would be incomplete as others began anew; and sequencing might be regarded as interesting, rather than as simply a difficulty to overcome.)

Given this sequencing, and the general method by which Sraffa proposes to compute prices, the prices of worn equipment must be computable. In fact, since *ownership* has been treated as irrelevant up to this point, no particular reason holds for Sraffa *not* to proceed as if any and all industries change hands at the end of each productive cycle.

Sraffa asserts that handbooks of commercial arithmetic usually calculate depreciation as

$$\frac{r \cdot (1 + r)^n}{(1 + r)^n - 1} \cdot p_{m_0}$$

where p_{m_0} is the original price of the equipment, r is the general rate of profit, and n is the life of the equipment.[12, §75] This valuation is of course an accounting convention, used as a heuristic. (We should perhaps note that, in commercial handbooks, r is actually the prevailing rate of *interest* — which outside of equilibrium may or may not equal the general rate of profit — and n is the *expected* life of the equipment.)

Sraffa shows that, given that equipment maintains constant efficiency from one period to the next until it simply fails at the end of an n th period, and considering a process with just one sort of multi-period depreciating equipment, exactly the formula above emerges from his approach. Since the depreciated equipment is conceptualized as a product of its industry, it is priced according to equations

$$\begin{aligned} p_i \cdot c_i(c_{1,i}, c_{2,i}, \dots, c_{k,i}, m_t, L_i) + p_{m_{t+1}} \cdot m_{t+1} \\ = w \cdot L_i + (1 + r) \cdot \sum_{j=1}^k (p_j \cdot c_{j,i}) + (1 + r) \cdot p_{m_t} \cdot m_t \end{aligned}$$

where m_t is equipment that has been used for t productive cycles, and p_{m_t} is the price of such equipment. Algebraically, these yield

$$\begin{aligned} p_i \cdot c_i(c_{1,i}, c_{2,i}, \dots, c_{k,i}, m_t, L_i) \\ = w \cdot L_i + (1 + r) \cdot \sum_{j=1}^k (p_j \cdot c_{j,i}) + \frac{r \cdot (1+r)^n}{(1+r)^n - 1} \cdot p_{m_0} \cdot m_0 \quad . \end{aligned}$$

Thus the annualized charge for the equipment is as given in the handbooks,[12, §76] with p_{m_0} presumably coming from a solution to the full system.

Sraffa continues

While the two methods give the same result in the extremely simplified case of constant efficiency to which both can be applied, the advantage of the joint-production-equations method is that it is not restricted to that case but has general validity.[12, §77]

But, unless one counts using a commercial handbook as a *method*, we're simply talking about the same formula, reached by unknown method in one case. Sraffa's method can of course abandon the assumption of constant efficiency (as might the method used to generate the formula in the handbooks), but his method has general validity only to the extent that its various *other* assumptions have general validity.

He labors the point that the book values of worn machinery must differ across applications if the machinery is worn differently across those application.[12, §78] And, in the special case of durable commodities, he illustrates the general point that his method of Reduction is not applicable.[12, §79]

Sraffa applies his argument from consideration of sub-systems to equipment that lasts beyond one productive period, and argues that again, in the absence of profits, the values of such equipment will be that of the labor that they embody.[12, §§80–81] But, again, the method of sub-systems fails account for the cost of maintaining resources not produced in surplus. The amount of equipment that exists at the end of each period of production may exactly equal the amount at the beginning, with the same portions at each stage of depreciation. A community is easily imagined that does not feel a need for a growing number of tractors, but the price of a tractor should not then be undefined or infinite. (I will explain why the method is more generally wrong in a later section.)

Sraffa notes that constant rates of depreciation on machinery of constant physical productivity are not compatible with a rate of profit other than 0; instead, as machinery ages its rate of depreciation must accelerate.[12, §82] He offers some example calculations, presumably based on his method of sub-systems.[12, §83] He notes that durable machinery that are basic commodities may participate in the Standard system by treating machinery of different ages as different commodities.[12, §84]

Land

Sraffa does not define what he means by “land”, but associates what he calls “rent” not only with charge for use of the indestructible properties of an area of soil but more generally with natural resources such as mineral deposits. Sraffa reiterates that these are not generated by a productive process, and thus cannot be counted amongst what he has called “basic” commodities, albeit that “they are the converse of commodities which, although produced, are not used in production”. (His peculiar way of treating labor has likewise made it other than a basic commodity, yet part of the Basic system.) He further insists upon the doctrine that the incidence of taxes on rent is borne entirely by landlords.[12, §85] I reiterate that this claim hangs not only upon some dubious assumption

or assumptions about technical constraints or about preferences, but also upon an unexplained theory of the mysterious money market.

Sraffa asserts that, with *corn* as the only agricultural product and with n different qualities of land in use, there will be n different methods of production for corn, “to which must be added the condition that one of the lands pays no rent”. He does not provide an explanation as to why some land would have a rent of 0; Ricardo famously argued that the least productive sort of land would earn a *rent* of 0, but his argument turned upon regarding the relevant properties of *land* as being indestructible or at least of a conception of *rent* (in the context of land) exactly and only as payment for use of indestructible properties.[9, Ch 2] Possibly Sraffa is using “land” to mean an instantiation of indestructible properties, but if so then he ought to declare as much, and he does not.

He notes that

Only the process that produces corn on the no-rent land can enter into the composition of the Standard system, since the no-rent land itself is eliminated from the equation, along with all other ‘free’ natural resources which, although necessary to production, are not reckoned among the means of production.[12, §86]

but, without some better reason for presuming the existence of such land, the need of the Standard system for no-rent land (if land is to enter into it) would simply argue that the Standard system cannot be embodied.

His price equations,[12, §86]

$$\left[\begin{array}{l} p_{corn} \cdot c_{corn,j} \\ = (1+r) \cdot \sum_{i=1}^k (p_i \cdot c_{i,corn,j}) + w \cdot L_{corn,j} + \rho_j \cdot \Lambda_j \end{array} \right] \forall j \in \{1, 2, \dots, n\}$$

(where $c_{corn,j}$ is corn produced with the j th sort of land, $c_{i,corn,j}$ is the quantity of the i th commodity used in that process, $L_{corn,j}$ is the quantity of labor there employed, Λ_j is the quantity of that land used, and ρ_j is the rent that it earns), show that he is again imagining production functions with single outputs,

$$c_{corn,j} (c_{1,corn,j}, c_{2,corn,j}, \dots, c_{k,corn,j}, L_{corn,j}, \Lambda_j) \ni j \in \{1, 2, \dots, n\} .$$

As well as imposing a condition that land of some sort should have a rent of zero, he imposes a condition that no land should have a negative rent,[12, §86]

$$[(\rho_j = 0) \exists j \in \{1, 2, \dots, n\}] \wedge [(\rho_j \geq 0) \forall j \in \{1, 2, \dots, n\}] .$$

Given that but one commodity is to be produced with land, and that land is of one quality, if it is not scarce then the process least costly in other commodities will be used.[12, §88] However, if land is scarce, then to some extent a process with greater yields, at greater cost, will be used in lieu of unavailable land. Thus the scarcity of land expresses itself in the system of pricing equations with a duality of equations. Their coefficients in construction of the Standard system will have opposite signs such as would cause land itself to be eliminated.[12, §87] However, Sraffa makes no note of cases in which a scarcity of land is offset

by using a more expensive method of production, and that employment meets present desire for the product exactly when applied to all available land, so that only one method is used. In this case, *two* variables, p_{corn} and ρ , correspond to *one* equation.

Instead of attending to that unacknowledged possibility, Sraffa moves on to imagine cases in which the production from land is kept continually rising with two methods of production always in use, but not the same two. Just as a more expensive method with higher yields completely displaces a less productive method, yet another method is introduced, still more expensive but with still higher yields.[12, §88] A problem for such considerations is that so much, up to this point, has depended upon an assumption that any surplus would be *consumed*, rather than reinvested.

Sraffa asserts that more complex cases may be decomposed into combinations of cases in which each product is generated on land of n sorts with as many distinct processes and with the poorest sort of land receiving no rent, and cases in which each product is generated upon just one sort of land with two distinct processes. In the case where land is of just one quality but multiple products are generated, he says

It may however be noted that only for *one* of the crops would two separate methods of production be compatible; for the rest, the number of processes would have to be equal to the number of products.[12, §89]

But he provides no explicit justification of this requirement. It seems that the number has to be equal for no better reason than that his other assumptions are inadequate for solution if it is not.

Sraffa draws attention to the point that multiprocess production necessitates negative coefficients in construction of the Standard system just as did multi-product processes. He proposes to reconstruct the notion of a *system* to be one of a set of industries or of methods of production equal in number not just to the number of different commodities *produced* but additionally to any unproduced commodities used in production.[12, §90] I confess that I don't know why he feels the need to labor this point.

Before leaving the subject of rent, Sraffa observes that machines whose production has been ended but whose use has not been exhausted may be regarded as receiving something like a rent; and that, like land, they are excluded from the category of basic commodities.[12, §91]

Switch in Methods of Production

Sraffa revisits the subject of changing selection of methods of production as the rate of profit r is exogenously changed. Initially, he also returns to the assumption that each productive process generates only one commodity.

He first considers the case of a non-basic product, and argues that a producer who builds a new plant will find it most profitable to build that which produces at whatever has become the least price.[12, §92] Sraffa has previously insisted

that there were but one rate of profit,[12, §4] but makes reference to “the general rate of profits” (underscore mine)[12, §92] as if there might be others. However, he offers no explanation as to how his model were to be generally adjusted to allow for other than a uniform rate of profits. One might imagine that a producer could make greater profit, without enjoying a greater *rate* of profit, by increased volume of the product in question or by redirecting freed resources to other production, apparently also of non-basic commodities.

When the product is basic, a change in its method of production implies a change of the Standard system corresponding to that under which it is produced. At a rate of profit at which exactly two methods of production produce at the same cost, two Standard systems result in the same set of prices and the same wage-rate. But, with any departure from an intersection of systems, different values for the maximum rate of profit apply (until and unless to a different intersection of the same systems). Sraffa is concerned to show that calculations within one of these Standard systems will not indicate a different choice about which technology to use than calculations within the other.[12, §93] Standard systems were constructed for exhibiting a relationship between wages and the rate of profits; it was not shown that those making decisions on when to switch technologies somehow employ a Standard system to do so. And, in the real world, changes in the employment of technologies seem neither necessary nor sufficient for observed changes in the monetary unit.

But, on a presumption that the relevant numéraire will be determined by the Standard system corresponding to the system prevailing, he seeks to show that the same decisions about which technology to employ will be made, regardless of which Standard system is thought to be in use at rates of profit in which multiple systems are possible. Thus, if at a rate of interest in which a system I and a system II are possible, and the calculations within system I say to remain within that system, then the calculations within system II say to switch to system I, and *vice versa*.

Sraffa begins referring to *uses* that he calls “basic” and “non-basic”. [12, §93] Unfortunately, he does not define either. Previously, the term “basic” referred to sorts of commodities, not to sorts of uses; the reader is left to infer or to guess what Sraffa now means. Under his initial definition of “basic”, a commodity were *basic* if it were used in the production of all commodities (including that of itself). A basic *use* might then simply be use in production; but he suggests copper as a good for which there might be *non-basic* use, and I don’t believe that he were imagining people furnishing their homes with lumps of it. Under his later definition, a commodity were *basic* if it could not be algebraically eliminated from the system of equations corresponding to the production functions of an economic system. The reduced system of equations, which he calls a “Basic” system, no longer shows actual *uses*. The Basic system cannot even be embodied unless economies of scale are co-incident (and then *in vacuo*); and, in any case, what would then be its uses are not generally the same as those of the original system.

Sraffa asks us to imagine, momentarily, first a system in which in which one method is used to produce a commodity strictly for “basic” use while another

method is used to produce the very same commodity strictly for “non-basic” use, and then a system in which the first method is used to produce the commodity for “non-basic” use and the second for “basic” use.[12, §93] But the multiple systems that each could potentially prevail at some rates of interest are incompatible, not withstanding that he gives quantities of this otherwise identical commodity distinct *labels* based upon which method were used to produce it, as if he might have his copper and eat it too. He is attempting to offer compared economic interpretations of two composite economic systems each of which is impossible under his prior assumptions. Once a contradiction is allowed, anything is logically possible.

The intention of committing product from one process to “basic” use and from the other to “non-basic” use is somehow to involve one process in the calculation of the numéraire and to exclude the other process yet then figure its accounting costs from the results of the first calculation. But one does not need actually to employ a productive process to compute its accounting costs under some set of prices; and a demonstration that the same decision would be made under either accounting should not involve us in peculiar yet undefined terms, and certainly not in self-contradiction.

Sraffa notes that, as any two systems might have more than one rate of profit r at which either might prevail, so that production might switch back and forth between them as the rate of profits were changed, nothing guarantee holds that, as the rate of profit is increased, switches will consistently be to systems of higher maximal profit R . However, in those cases where either system might prevail at only one rate of profits, as the rate of interest r is increased, any switch must be to a system of higher R . [12, §94] Also, in a system of single-product processes, as the rate r is increased, then w is decreased relative to the price of every commodity, within any system, exactly because these price-sets are intrinsically intra-systemic. [12, §95] Nothing is thereby said about how a switch in systems, in response to a change in r , would affect wages viewed inter-systemically.

Quietly presuming that no commodity’s price falls faster than does the wage rate, [12, §96 fn 1] Sraffa argues that, in the case of multi-product functions, though the wage rate must fall within any system given an increase in the rate of profit, if wages across systems are expressed in the same standard then the system that allows the highest wage for any given rate of profits will be the most profitable system and hence that system which is selected. The process superseded in response to a change in the rate of profit r would be that not present in the selected system, [12, §96] though I’m not sure why this last, tautologic point is thought to be of interest.

And with that, *Production of Commodities by Mean of Commodities* abruptly ends.

Additional Discussion

Some matters upon which I touched during the walk-through merit consolidated reiteration or other further discussion.

The Obscured Presence of the Subjective

Nowhere in *Production of Commodities by Mean of Commodities* does Sraffa actually claim to have completely eliminated a rôle for the subjective in price formation. Yet, in his famous clash with v. Hayek, Sraffa referred to “the imaginary accomplishments of the ‘subjective method’”, [11, p 47 fn 1] suggesting that he thought that it had no real accomplishments; *Production of Commodities* is sometimes claimed by others to have shown how prices are determined without subjective considerations. (For example, Sinha writes “Within this frame, the prices of commodities must take certain values independently of any psychological factors of agents.” [10, p 309]) In fact, the subjective is present, but everywhere that it enters into his model, its participation is *obscured*.

It is present in his first model, of the subsistence-level economic system, in the selection of prices that support subsistence instead of leading to extinction; it is obscured by failing to acknowledge the possibility of extinction, nor the desirability of an explanation of how a market would settle upon prices that are just right for subsistence. In his second model, that with surplus production, the subjective enters into the willingness of profit-seekers to engage in any activity so long as the same rate of pecuniary profit is realized; it is obscured because this simplifying assumption is so commonplace as to be accepted thoughtlessly. Also in his second model, if one does not assume what amount to co-incident returns to scale in the production of basic commodities, then assumptions about the desire for goods being exhausted must underlie his claim that a change in the price of a non-basic commodity cannot affect that of a basic commodity; the possible presence of the subjective is obscured by his ignoring the question of potential growth in the supply of basic commodities until most readers will have quite accepted his proposition about the relationships amongst prices.

Perhaps most importantly, in his third and subsequent models, those with a separated labor component, workers don’t down tools when paid nothing and, when given the option of the whole surplus, they don’t decide to substitute leisure for any part of that; they would always *prefer* to work at some level constant across all wage rates. This remarkable preference (upon which so very much depends) is obscured by its invariance combined with the effect of giving the total possible quantity of labor a value of 1, so that its presence becomes *implicit* rather than participating as an explicit ‘ L ’. Additionally, a uniform wage rate w implies a lack of concern for the non-pecuniary of much the same sort as does a uniform rate of profit r .

I cannot explain how Mr Sraffa could by accident at every one of these junctures disguise the subjective element.

On Returns to Scale

If truly all inputs were proportionately increased, then only constant returns to scale would make sense. However, in the real world, one is virtually never increasing all inputs proportionately; acknowledged inputs may be scaled proportionately, but some unacknowledged inputs are not. Returns to scale don’t

quite have to be constant to be co-incident; one may even imagine a production function of inconstant returns to scale that repeatedly intersects one of constant returns. None-the-less, if the imagined function does this at every point in which constant returns to scale would prove convenient, then the returns operationalize as constant, and it would be disingenuous to deny assumption of constancy without admitting assumption of convenient co-incident.

Although Sraffa denies making an assumption that returns to scale are constant for the production of all commodities, he repeatedly presumes for *basic* commodities that returns are co-incident. At many points in the book, this assumption could be abandoned were he to let go of imagining the Standard system as able to be embodied. But, given that he eventually allows for increasing production of a basic commodity (the tractor), so that it figures in the surplus, his claim that changes in prices of non-basic commodities cannot affect those of basic commodities will not hold unless returns in the production of those increasing commodities are co-incident. Thus, the implicit assumption of co-incident proves to be quite essential to the construction of the Standard commodity.

On the Relationship of Price to Cost

Sraffa offers models in which the price of every commodity is the price of that quantity of basic commodities that enter (directly or indirectly) into its production, plus a share of any surplus, which enters as a wage cost or as a profit claimed. The composition of the surplus may be chosen, but its net value is that of the Standard commodity, whose size is determined by the quantities of basic commodities and labor employed by the economic system, and these are treated as not chosen or as chosen by the setting of the rate of profit r (as this may determine which productive processes are used for basic commodities). In this context, Sraffa claims that the prices of basic commodities do not affect those of non-basic commodities, while the prices of non-basic commodities are affected by those of the basic commodities that enter into their production.

However, as any share to the workers and any share to the rentiers comes exactly and only from surplus production, if there were any *human* objective to the system then it were to produce the surplus. And when the rate of profit were set or reset, effecting a change in the Standard system and hence in the prices of basic products, that setting or resetting were exactly to change the surplus. The prices of basic commodities would thus indeed be affected by the prices of non-basic commodities. Further, in the real world, workers respond to a change in the prices of commodities by seeking more income or to vary leisure. Sraffa's model only allows them (perhaps) to struggle to change the rate of profit; only by quietly disallowing a change in the quantity of labor employed he does he disallow a change in the prices of non-basics good to affect those of basic goods.

Nowhere in these models are costs reasonably presented as the ultimate determinants of prices, either for basic or for non-basic commodities.

On the Method of Sub-Systems

While, because of an implied division by 0, the failure of the method of sub-systems to equate the price of a commodity with a wage-cost equivalent is most pronounced in the case of commodities that are not produced as surplus, the failure is not confined to such cases.

In the real world, things rot, cross-link, and corrode; the source of labor must be fed or it starves; and so forth. Plainly, a subsistence-level economic system cannot be idle; plainly, its resources at the end of each productive period are not simply identical to those available at the beginning, though they may be exactly equal in numbers; even many of whatever things that are truly unchanged needed resources to stay that way. Hence the literal absurdity of dividing total labor expenditure by net product.

Likewise for an economic system producing some one commodity as surplus; it is not just producing that commodity. If the resources of the economic system were all idled then, at the end of the unproductive period, there would be not merely an absence of the commodity that could have been produced as surplus but a reduction of some or all of the idled factors. Properly, those other things are co-products, consuming a share of the labor that Sraffa imagines goes only into the surplus product.

On Switching

Although there were two prior episodes in the Twentieth Century of marked controversy about the nature of capital and about its relationship to rates of interest,[1][2][3][4][5][6][7] the most familiar is the more recent “battle between the Cambridges”. Mr Sraffa’s noted contribution was in his demonstration that a change in the rate of interest (if we take this to be the same as the general rate of profit) could induce a switch in methods of production and thence a change in the composition and summed value of capital.

Yet, for most purposes, the mainstream of macroeconomic thinking still tends to treat capital as a great and often exogenous K . The mainstream is not wholly unaware of the problem; in the prior controversies and in the more recent dust-up, the champions of what was to become or had become that mainstream acknowledged that this model of capital weren’t strictly correct. But they believe that, for the purposes to which they put it, it were close enough. To show that reswitching *can* occur or even to show that it *does* occur is not the same thing as showing that its effects are sufficient to invalidate whatever conclusions may be drawn by ignoring such things.

I wish to return attention to the point that, just as land is not homogenous, and capital is not homogenous, neither is labor homogenous. Yet Mr Sraffa writes

We suppose labour to be uniform in quality or, what amounts to the same thing, we assume differences in quality to have been previously reduced to equivalent differences in quantity so that each unit of labor receives the same wage.[12, §10]

We are used, of course, to labor of various sorts being priced in some common currency, but likewise for capital of different sorts. The apparent quantification in terms of money no more implies the relationship supposed amongst sorts of labor by Mr Sraffa than it implies the relationship amongst sorts of capital that other economists suppose.

To generalize Mr Sraffa's models to acknowledge the heterogeneity of labor, one would distinguish amongst methods of production by the sorts of labor that they used; there would be different wages rates for different sorts. As r varied (presuming that it continued to be exogenously determined), there would be switches in the methods of production, and *these wages rates would not simply maintain fixed ratios each to the others.*

Yet he forges ahead, in an apparent belief that the violence done to reality will not overwhelm the potential insights to be gained, just as mainstream economists forge ahead in treating capital similarly. Such beliefs are almost certainly correct, within some limits. Unfortunately, those limits are unknown; if they are ever to be found then economics must be constructed without such approximations. If the limits are to be even themselves well approximated, it must be by more careful and very much more difficult construction.

In the mean time, that a great and often exogenous K is somehow worse than a great and exogenous L is unclear; and either is better than a 1.

Conclusion

To the extent that Sraffa's *Production of Commodities by Mean of Commodities* is not ignored, it is generally misunderstood. Were it understood, it would be more widely rejected; its conclusions depend upon a grossly unrealistic model of labor supply, upon co-incident returns to scale of all those products that factor directly or indirectly into the production of all products, and upon a method of sub-systems that has lost sight of the need to replace perishable factors.

In large measure, the book has not been better understood because its exposition is needlessly obscure. The underlying theory is mostly mathematic, but Sraffa avoids formal mathematic expression, and seeks to justify conclusions by what he regards as economic reasoning. Additionally, some readers have been entirely too eager to accept its conclusions (and what are believed to be its conclusions) or too averse to accepting those conclusions to attend to the difficulties of carefully considering the process by which the work is developed.

Any theory may be transformed into any other theory by process of revision; and so, as a logical matter, Sraffa's work is salvageable. But it would almost certainly be more efficient to start fresh.

Its point about the implications for capital theory of switches in methods of production is clearly of interest, but separable from the rest of the work, and not evidently damning to every use of assumed homogeneity in constructing theory. Only a far more careful development of economic theory and empiric determination of parameters would tell us just when that assumption would begin to lead to erroneous conclusions.

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